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Coupled states of flapping flags

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Abstract

Experiments concerning the oscillatory motion of thin flexible filaments lying in flowing soap films have shown particularly interesting phenomena. For example, when two such ‘‘flapping’’ filaments were introduced to the flowing soap film the filaments oscillated in-phase (asymmetry) or in anti-phase (symmetry) depending only on their mutual separation. We have been able to numerically simulate these experiments and provide some additional data on how the interaction changes when the filament separation is varied. Our results show that the fundamental frequency is dominant (single mode) and that the oscillation consists of an asymmetric system for small separations. For larger separations of about one half of the filament length, the fundamental frequency is again dominant although the system exhibits symmetric motion in this case. However, the filaments appear to pass through a domain for which the oscillation is possibly quasi-periodic for intermediate values of the mutual separation. In addition, initial investigations suggest that the frequency of oscillations of the system is greater by approximately 30% for larger separations than for smaller separations. We note that an increase of similar magnitude was also observed in experiment.

1. Introduction

Recent experiments (Zhang et al., 2000; Rutgers et al., 1996; Chomaz and Cathalau, 1990) have investigated the fascinating topic of thin flexible filaments lying in flowing soap films and these systems have been shown to demonstrate particularly interesting phenomena. Indeed, when two such ‘‘flapping’’ filaments were introduced to the flowing soap film then these filaments were seen to oscillate in-phase (asymmetry) or in anti-phase (symmetry) depending only on their mutual separation.

Interacting vortex wakes emanating from structures in close proximity have been extensively studied elsewhere [see, for example, Bearman and Wadcock (1973) in which it was shown that two wake configurations were possible]. In addition, experiments recently carried out and published in Nature (Zhang et al., 2000) showed that this double configuration could occur for *flexible* structures where the two wake arrangements could be induced by varying the separation distance of the filaments. We also note that flexible membranes or plates cantilevered from their leading edge are alternative models that have flapping flag-like characteristics and the interested reader is referred to articles by Watanabe et al. (2002a, b) and Tang et al. (2003) for more details.

We wish to present initial calculations that outline numerical results that show that it is possible to numerically simulate this strongly interacting nonlinear system by using a mixture of computational fluid dynamics and Lagrangian mechanics for the filaments. We assume that the fluid is incompressible, isothermal and laminar, with a viscosity that is Newtonian in form, and we use similar assumptions to the experiments for measuring the viscosity of soap films (Martin and Wu, 1995). We therefore solve the nondimensionalized version of the fluid flow momentum

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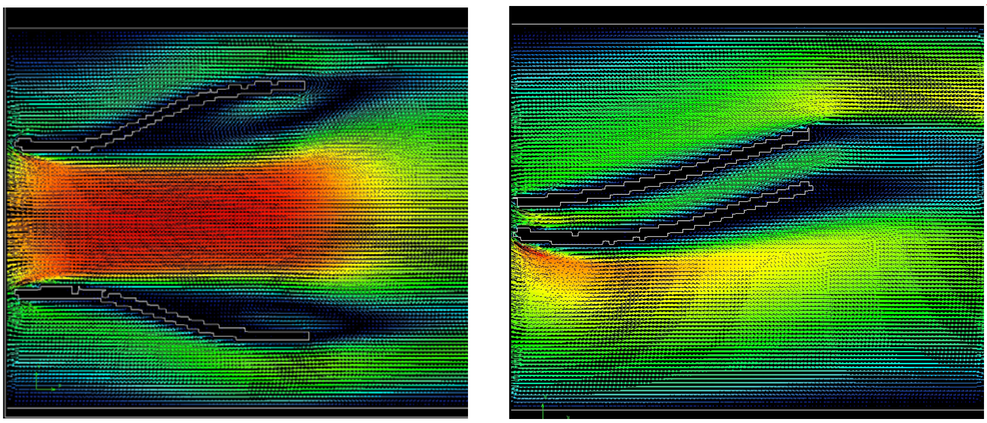
(Navier–Stokes) equations numerically. We therefore note that we are able to adequately simulate many of the phenomena seen in these experiments and that we are also able to provide some additional data regarding the nature of the filament interactions as the filament separation is varied. However, although the results presented in this article are clearly preliminary, we are still managing to capture much of the essential physics of this strongly coupled system. A more detailed analysis of our numerical solution will form the contents of another article.

We treat the filaments as each being composed of N equal-length, homogeneous, rigid elements. The elements are fixed to one another at their hinge (or fulcrum) points (Appendix A). The whole filament is therefore approximated by the form of an "N-tuple pendulum" in which each hinge has a positive spring stiffness coefficient, K_s, and a positive damping coefficient, C_s [see Fenlon and David (2001a, b) and Farnell et al. (2003) for more details]. Note also that Appendix B briefly outlines the values for parameters in the Euler–Lagrange equations for the filaments and which govern the internal stiffness and damping of the filaments and a nondimensional parameter, F, which describes the ratio of densities of the fluid and the filament. The leading edges of the filaments are fixed and are separated by a distance, D, nondimensionalized by the filament length (so $D = 1.0$ means a physical separation of one filament length). In order to model the motion of the N-tuple pendulum we form a system of equations using Lagrange mechanics (Fenlon and David, 2001a, b). These equations are integrated with respect to time using standard techniques in order to simulate the behaviour of each filament both simultaneously and independently.

We performed a "weakly" coupled calculation for the fluid and filaments elements of our simulation of this system (see Appendix A). In particular, the fluid was simulated at distinct time-steps and the fluid pressures, working on the upper and lower parts of each filament at a particular time-step, were found. We note that each filament was treated independently with the equations of motion for each filament integrated separately and that the only interaction between the filaments is therefore due to the flowing fluid between them. It is important to note that forces acting on the filament consisted entirely of pressure (normal) forces and that viscous drag was not imposed upon the filaments, although viscous action was taken into account in the determination of the fluid equations. A full account of the computational algorithm for a single filament in a flowing soap film is given in an article by Farnell et al. (2003) and we utilize the same approach here. Clearly, experiments concerning oscillating filaments in flowing soap films, and numerical simulations of them, yield much information (and provide insight into) the manner in which three-dimensional flags may flap. We shall now describe our results for the flapping states of the filaments, which we note are coupled via the intervening fluid.

2. Results

We consider the case in which there are two filaments, each placed side-by-side in our fluid domain as shown in Figs. 1a and b. We now vary D, the separation distance of the two filaments at their fixed leading-edge points. In Fig. 1a we



(a) (b)

Fig. 1. (a) Filament position and velocity vectors plotted for typical profiles for two filaments of mutual separation $D = 0.5$. Note that $\mathrm{Re} = 500$ and that the filaments demonstrate a stable oscillating mode. The filaments are clearly oscillating in anti-phase (symmetrically). (b) Filament position and velocity vectors plotted for typical profiles for two filaments of mutual separation $D = 0.1$. Note that $\mathrm{Re} = 500$ and that the filaments demonstrate a stable oscillating mode. The filaments are clearly oscillating in-phase (anti-symmetrically). (Note that the nondimensional peak velocity in the figures above, indicated by red, is approximately of magnitude 3, a higher velocity than green and green a larger velocity than blue, etc. The fluid velocity at the walls is zero.)

may observe that the filaments oscillate and they appear to be "locked" in anti-phase (symmetry) for the case of $D = 0.5$. By contrast, in Fig. 1b we find that they are locked in-phase (asymmetry) at $D = 0.1$. Furthermore, our numerical results suggest that there appears to be a change in behaviour between $D = 0.3$ and 0.2 from being in anti-phase to being in-phase. We may also see that vortices are shed from the trailing edge of both the upper and lower filaments with equal strengths but of opposite sign in Fig. 1a, as was seen in experiment.

In all of our simulations we note that the filaments never touch each other even though we do not impose this as an external condition on the system. They are thus held apart purely due to the pressure forces in the intervening fluid. We are able to analyse the interacting behaviour of the two filaments in much more detail by considering the displacement normal to the fluid flow (y-values) of the trailing edges of the upper and lower filaments with respect to time and this is plotted in Fig. 2 for two values of $D = 0.5$ and 0.3. The initial configurations for the two filaments are quickly subsumed into the stable oscillating mode.

In Figs. 2a and b, we may see that the filament oscillations are stable with respect to time in the simulations for $D = 0.5$ and 0.3, and that the maximal y-values of the trailing edge of the upper filament correspond to minimal y-values of the trailing edge of the lower filament (and vice versa) with respect to time. This is a clear indication that filaments are phase-locked in anti-phase (symmetric). In Fig. 3, the y-values of the trailing edges of the filaments with respect to time for $D = 0.1$ again show that the upper and lower filaments are in stable oscillatory modes. By contrast with the $D = 0.5$ case above, the filaments are now clearly phase-locked in-phase (anti-symmetric) with each other for $D = 0.1$ as minimal values for the upper filament correspond to minimal values for the lower filament, with respect to

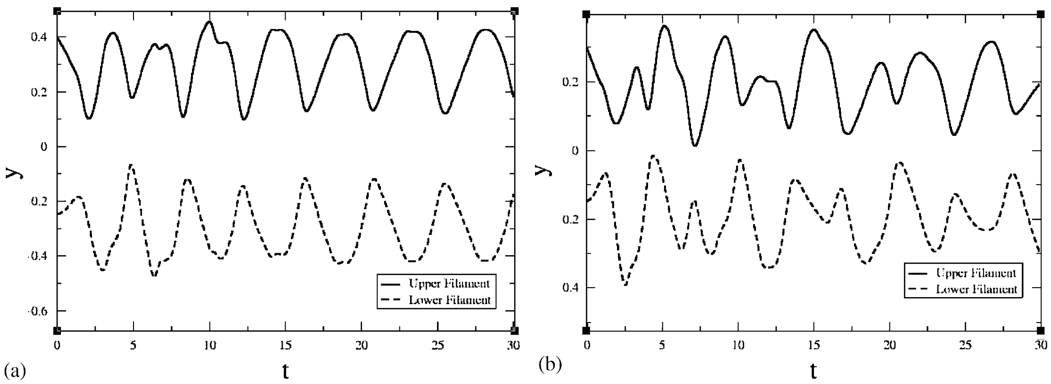


Fig. 2. y-Values of the trailing edges of the upper and lower filaments plotted with respect to time for two filaments for a separation of the leading edges of (a) $D = 0.5$ (left) and (b) $D = 0.3$ (right), respectively.

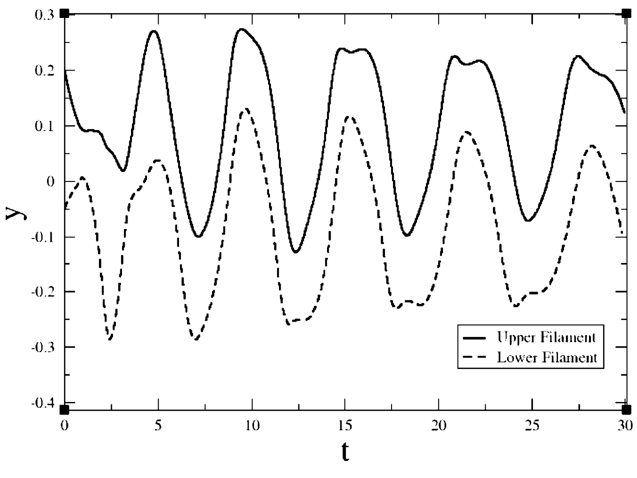


Fig. 3. y-Values of the trailing edges of the upper and lower filaments plotted with respect to time for two filaments oscillate in anti-phase with each other for a separation of their leading edges, $D = 0.1$.

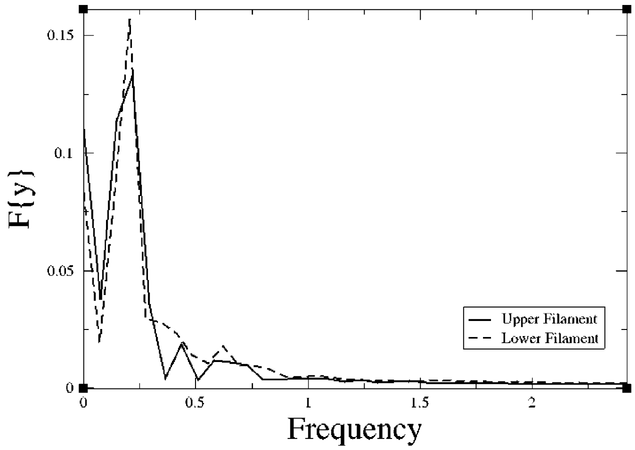


Fig. 4. Fourier transform of the y-values of the trailing edges plotted with respect to frequency (nondimensional) for $D = 0.1$.

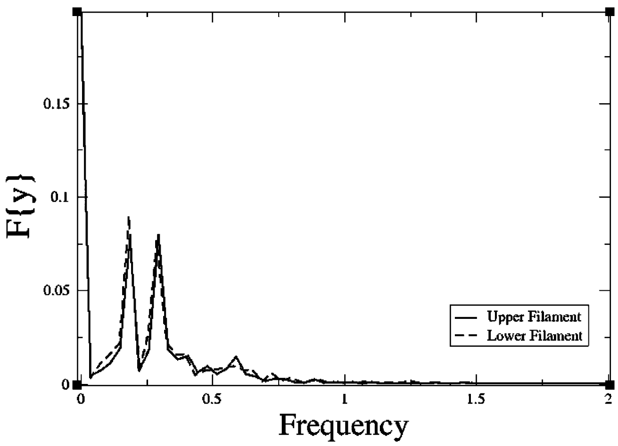


Fig. 5. Fourier transform of the y-values of the trailing edges plotted with respect to frequency (nondimensional) for $D = 0.3$.

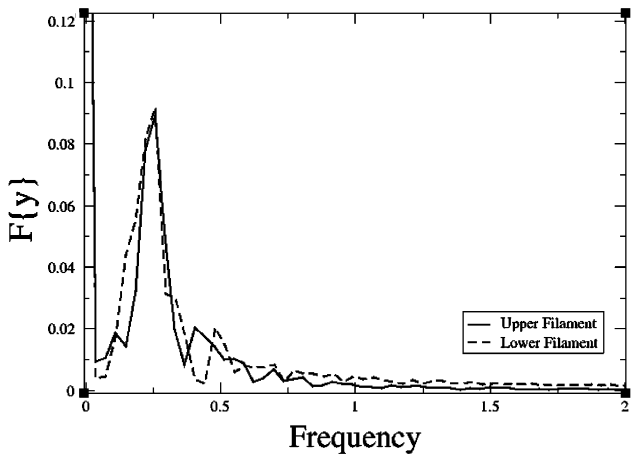


Fig. 6. Fourier transform of the y-values of the trailing edges plotted with respect to frequency (nondimensional) for $D = 0.5$.

time, and maximal values for the upper filament correspond to maximal values for the lower filament. We conclude that we have now adequately simulated the other "mode" seen in experiment for the flapping silk filaments.

We were able to force the system from one mode to the other by changing only their separation, D. The simulations show that the regular pattern of oscillation for $D = 0.5$, 0.4, and 0.1 is much reduced for $D = 0.3$ and 0.2. Indeed, the magnitude of the y-values of the trailing edges of the filaments at the peaks and troughs appear not to be constant for $D = 0.3$ and 0.2, and this might indicate that we are approaching a fundamental change in the behaviour of the system.

Figs. 4–6 show Fourier transforms of the y-values for $D = 0.1$, 0.3 and 0.5 respectively with respect to the nondimensional frequency. (Note that the peak at zero frequency is due to an average nonzero value for the y-displacements of the trailing edges because the filaments have a nonzero mutual separation, and so this peak may be safely ignored.) The interesting phenomenon lies in Fig. 5 where there two modes of equal "amplitude" exist. This in contrast to those results for small and large values of D in which only a single mode is dominant. The case for $D = 0.3$ may well be an example of the quasi-periodic oscillations found for coupled cylinders in a flowing fluid (Peschard and Le Gal, 1996). The simulations also show that the dominant frequency increases by approximately 30% for $D = 0.1$ compared to 0.5, as may be seen in Figs. 4 and 6. This compares very favourably to the experimental result (Zhang et al., 2000) of an increase of 35% as the separation is commensurately increased. We note however that the magnitude of the fundamental mode of the Fourier transform is smaller for $D = 0.5$ in Fig. 6 than for $D = 0.1$ in Fig. 4.

Animations generated from the numerical time-dependent results show that, for the case where a single wavelength is present along the filament (the fundamental mode), the effective bluff body shape of the leading edge of the filament induces boundary layer separation as normal. Two small recirculation zones exist and if the body were nondeforming then these zones would be shed periodically. However, since the filament is deformable, then as one of the recirculation zones starts to grow the asymmetric pressure profile (measured between upper and lower surfaces of the filament) induces a curvature in the filament itself. This allows two competing phenomena. Firstly, the recirculation zone on the side of concave curvature grows. This induces further curvature of the filament. Secondly the recirculation zone on the opposite side is effectively trapped by the convexity of the deforming filament. Hence, only one vortex per filament is allowed to develop at a time.

This vortex is convected down the filament length at the wavespeed of the filament curvature. During this convection period the curvature of the filament "follows" the motion of the recirculation zone, essentially a "lock-in" phenomenon (Anderson et al., 1998). However, although the Strouhal number can be used to represent vortex-induced vibration for nondeformable bodies this is not the case here. The vortices are enhanced by the curvature of the filament (which is a function of the stiffness and damping parameters in the Euler–Lagrange equations for the filaments), which are in a circular fashion created by the existence of the vortex itself. Hence, we can see that energy is being given to the filament from the fluid by virtue of the generation and enhancement of the recirculation zones. This phenomenon is somewhat similar to that seen by where oscillating foils have their highest efficiency when a leading-edge vortex is convected downstream and coalesces with the trailing edge vortex to cause a reverse Kármán street, indicating that the fundamental flapping mode may correspond to the minimal energy state.

3. Conclusions

We have presented initial and preliminary calculations which support experimental evidence which suggests that when two such flapping filaments were introduced to the flowing soap film the filaments oscillated in-phase (asymmetry) or in anti-phase (symmetry) depending only on their mutual separation. A more detailed analysis of the stability of our numerical solution will form the contents of another article. We have seen however that there appears to be change in behaviour of this system for a mutual separation of the filaments of value $D = 0.2$ and 0.3 (nondimensional values with respect to of the length of the filaments). At this point, we also note that two peaks in the Fourier transform of the y-values of the trailing edges of the filaments were observed. This is in stark contrast to the case for larger and smaller separations in which only a single dominant (fundamental) frequency was observed. Furthermore, the simulations also show that the dominant frequency increases by approximately 30% for $D = 0.1$ compared to 0.5. We note that this result is in excellent agreement with experiment (Zhang et al., 2000), which predicts a 35% increase in the frequency of oscillation of the filaments as the separation is increased. Clearly, we are capturing much of the essential physics of this strongly coupled system. The magnitude of the peak for the fundamental frequencies with respect to Fourier transforms of the y-displacements of the trailing edges is found to decrease for $D = 0.1$ compared to $D = 0.5$.

We may also extend our simulations to different scenarios for the two filaments in future calculations. For example, we may wish to place the filaments in different positions (e.g., one filament downstream of the other) or to introduce more than two filaments into the two-dimensional fluid domain. In principle, this ought to be straightforward. Indeed, we plan to make a full survey of the stability of our numerical solution, as mentioned above, and also to "map out" the regions of stability of the flapping states for the simulations presented here with respect to the parameters which govern the behaviour of the fluid and the mechanical structures (i.e., the filaments). These parameters are the Reynolds number and the stiffness and damping terms in filament Euler–Lagrange equations. We might also determine the kinetic and potential energies of this system as a function of time, and we expect these quantities to vary with time because this is a nonconservative system. Indeed, it would be interesting to observe if the amount of energy extracted by the filaments from the fluid increases or decreases as we reduce their mutual separation.

Finally, there are also many strongly coupled fluid/structure systems of biological interest and numerical simulations may help in understanding the fundamental processes at work in them. A good example is that of artificial heart valves, which we note have been simulated using analogous techniques. Indeed, it might be impractical, expensive, and/or clinically undesirable to observe and probe them in vivo, and numerical simulation might provide a very useful alternative tool. Indeed, the search for a "perfect" artificial heart valve is still ongoing and this is another strong motivator in this area. Also, differently shaped drums can, in principle, create an identical frequency response when struck and so we might ask: "does sound have a shape?" This question might be addressed using such techniques as those presented here in the context of biology, for example, by studying the frequency response of the *lamina spiralis* in the ear.

Appendix A. Numerical method

As a detailed explanation of the numerical procedure, including a discussion of the fluid mesh and an explicit description of the Euler–Lagrange equations for the filaments, is given in the article by Farnell et al. (2003), a brief outline of the numerical method is given here only. We note however that the computational domain is shown in Fig. 7 and consists of a simple two-dimensional rectangular area whose (nondimensional) length is H and width h with fluid flowing from left to right. The length of the filament is 1 (in nondimensional units), although the width h of the fluid domain is set slightly larger at 1.25 (nondimensional units) in order to give the filaments sufficient space to oscillate with large amplitude. The length of the fluid domain was set at $H = 15$ units and we note these setting are fully consistent with experiment.

For the "real" system, we note that the actual, dimensional length of the filament is given by L, that the maximum fluid flow at the inlet is given by U_∞, and that the kinematic viscosity is given by ν. We nondimensionalize the system by scaling by these variables by "characteristic" values, such that, for example, length is scaled by a factor of L and time is scaled by a factor of U_∞/L.

The fluid is assumed incompressible, of constant density, and Newtonian. The flow is treated as being isothermal and laminar, and we solve the time independent Navier–Stokes equations for the Reynolds number, $\mathrm{Re} = LU_\infty/\nu$, given by

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \mathrm{Re}^{-1}\,\nabla^2\mathbf{u}. \quad (1)$$

In order to simulate the two-dimensional soap film flow, no-slip boundary conditions are placed on the top and bottom sides of the computational domain as shown in Fig. 7. The inlet (left-hand end) has imposed a pressure boundary condition P_i and similarly for the outlet (right-hand end) P_o. The pressure difference $P_i - P_o$ is set to ensure that the velocity is $\mathcal{O}(1)$.

We treat the filament as being composed of N equal length, homogeneous, hinged filament elements. Thus, the whole filament (or flag) thus assumed to be approximated by a form of an N-tuple pendulum. Each hinge has a positive spring

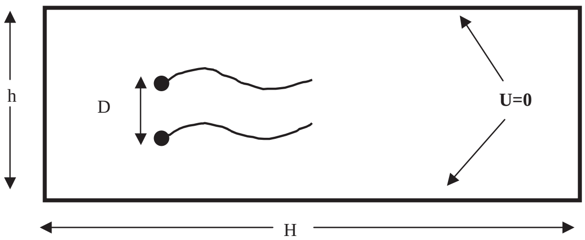


Fig. 7. Computational fluid domain. No slip boundary conditions are imposed on the upper and lower "walls" of the fluid domain. The fluid is made to flow from the "inlet" on the left to the "outlet" on the right by applying an appropriate pressure differential between them.

stiffness coefficient and a positive damping coefficient associated with it. We may obtain relevant Lagrangian for the filament and the Euler–Lagrange equations may be determined analytically. The position and velocity of each of the filaments are imposed as moving boundary conditions on the fluid and again a computational fluid dynamics calculation was performed in order to determine the fluid properties at the new time-step. This process was performed repeatedly in order to simulate the behaviour of the two filaments over a sufficiently large time in order to observe the asymptotic behaviour of the system. We therefore perform a weakly coupled calculation for the fluid and filament elements of our simulation of this system. Initial conditions for the simulation were that the upper filament makes a small positive angle to the horizontal and the lower filament lies along the horizontal axis. Initial configurations therefore do not favour any particular mode (i.e., in-phase or in anti-phase) for the pair of filaments.

Note that the fluid mesh is chosen such that there are always at least 3 mesh nodes within the filament thickness. This thickness is chosen here to be constant and also to be consistent roughly with experiment. We found that a filament thickness of 0.04 of the length of the filament was found to be adequate for our purposes. We note that refining the mesh whilst keeping the thickness of the filaments constant did not seem to change our results qualitatively very much. Indeed, the overall accuracy of the Runge–Kutta method used to simulate the filaments and the accuracy of the CFD calculation were set to be small enough that they also did not affect our results qualitatively by reducing them still further. However, we plan to make a full investigation of the numerical accuracy of our results and a full mapping of the range of stability of our simulations with respect to the parameters which govern the behaviour of the filaments and the fluid (such as the filament thickness and the fineness of the fluid mesh, the stiffness and damping coefficients in the Euler–Lagrange equations, and the Reynolds number for the Navier–Stokes equations). These calculations will form the contents of another article, and, although the results presented in this article are indeed preliminary, we may clearly see that we are capturing much of the essential physics of this strongly coupled system.

Appendix B. Simulation parameters

The values for the parameters which govern our simulation are discussed in the article by Farnell et al. (2003), although we now also present a brief outline of the numerical values of these parameters. As in experiment (Zhang et al., 2000), we chose a value specifically for the term which governs the internal stiffness of the filament, K_s, which produced a single wavelength with respect to the length of the filament. We also found that reasonable results were obtained for values of internal damping coefficient of the filament, C_s, which were of approximately the same or smaller magnitude than K_s. In this manner, we obtained a value for the stiffness coefficient of $K_s = 0.013$ and adequate results were obtained for $C_s = 0.0025$. We also note that the ratios of the densities of the fluid to the density of the filament may be adequately described by a single nondimensional parameter, F, given by

$$F = \frac{\rho_f L}{\sigma} \tag{2}$$

and where ρ_f is the $(2 - \mathrm{D})$ density of the fluid and σ is the linear density of the filament. Please note that we chose "typical" values for F and for the Reynolds number of $F = 1$ and of $\mathrm{Re} = 500$, respectively, which were fully consistent with the experimental situation, and that a detailed explanation of the nondimensionalization procedure (and thus the numerical values of the nondimensionalized filament element masses and lengths) and also a related discussion of a single filament resonant frequency is given in the article by Farnell et al. (2003).

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